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OPTIMIZATION OF ELECTRODYNAMIC ACCELERATION REGIMES FOR CYLINDRICAL CONDUCTORS

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At the present time electromagnetic accelerators which use the action of an impulsive electromagnetic field on a current-carrying conductor appear to be promising devices for the study of high-speed collisions. In the regime using separate sources for the accelerating magnetic field and the current in the conductor being accelerated it is possible to bring cylindrical conductors up to velocities exceeding 12 km/sec [1]. Acceleration regimes have been calculated previously [2] assuming independence of the current density in the conductor from the accelerating magnetic field. However, as analysis of transient electromagnetic processes occurring in the interaction of an impulsive electromagnetic field with a cylindrical conductor shows [3], the maximum current density, limited by heating conditions, depends significantly on the induction of the accelerating magnetic field. In the present study we will analyze regimes for electrodynamic acceleration of cylindrical conductors with consideration of diffusion of both the intrinsic and the external impulsive magnetic field within the conductor.

We will use an idealized two-dimensional calculation model in which an infinitely long conductor with axial current i is located in a homogeneous transverse accelerating magnetic field with induction B. We will assume that the current and magnetic field induction vary with time as follows:

$$B = B_0[1 - \exp(-t/T)],\ i = i_0[1 - \exp(-t/T)]\eta(t - t_0),$$

where $\eta(t - t_0)$ is a unit function [4].

Such current forms can be realized with power supply from high-Q inductive supplies with time constants much greater than the acceleration time [5]. Similar forms can be obtained from capacitive supplies with active load switching [6]. Introduction of a delay time t_0 permits more complete use of the magnetic field induction, which, as will be shown below, allows attaining additional velocity in some cases.

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Assuming the problem to be linear, we apply the Duhamel integral [4] to be the expressions of [3]. Taking as the criterion of admissible heating the "current integral" $I=\int_0^t j^2 dt$ [6], we find an expression for the mean density of the axial current limited by heating conditions in relative units:

$$j_0^* = \left[-4\beta\alpha_2 + \sqrt{16\beta^2\alpha_2^2 - \alpha_1(16\beta^2\alpha_3 - 1)}\right]\Big/\alpha_1 \tag{1}$$

where

$$\alpha_1 = \int_{\tau_0}^{\tau_1}\left\{\left[1+\sum_{m=1}^{\infty} M_m e^{-\xi_m^2(\tau-\tau_0)}\right]\left(1-e^{-\frac{\tau_0}{T_1}}\right)+e^{-\frac{\tau_0}{T_1}}\left\{\left(1-e^{-\frac{\tau-\tau_0}{T_1}}\right)+\sum_{m=1}^{\infty} L_m\left[e^{-\frac{\tau-\tau_0}{T_1}}-e^{-\xi_m^2(\tau-\tau_0)}\right]\right\}\right\}^2 d\tau;$$

$$\alpha_2 = \int_{\tau_0}^{\tau_1}\left\{\left[1+\sum_{m=1}^{\infty} M_m e^{-\xi_m^2(\tau-\tau_0)}\right]\left(1-e^{-\frac{\tau_0}{T_1}}\right)+\right.$$

$$\left.+e^{-\frac{\tau_0}{T_1}}\left\{\left(1-e^{-\frac{\tau-\tau_0}{T_1}}\right)+\sum_{m=1}^{\infty} L_m\left[e^{-\frac{\tau-\tau_0}{T_1}}-e^{-\xi_m^2(\tau-\tau_0)}\right]\right\}\right\}\left[-\sum_{n=1}^{\infty} N_n\left(e^{-\frac{\tau}{T_1}}-e^{-\xi_n^2\tau}\right)\sin\varphi\right]d\tau;$$

$$\alpha_3 = \int_0^{\tau_1}\left[\sum_{n=1}^{\infty} N_n\left(e^{-\frac{\tau}{T_1}}-e^{-\xi_n^2\tau}\right)\sin\varphi\right]^2 d\tau;\quad M_m = \frac{J_0(\varepsilon\xi_m)}{J_0(\xi_m)};$$

$$L_m = M_m/(\xi_m^2 T_1 - 1);\quad N_n = J_1(\varepsilon\xi_n)/[J_1(\xi_n)(\xi_n^2 T_1 - 1)];$$

$$j_0^* = j_0/j_1;\quad \beta = B_0/B_1;\quad \tau = t/t_1;\quad \tau_0 = t_0/t_1,\quad T_1 = T/t_1;$$

$$\varepsilon = \frac{r}{a};\quad j_0 = \frac{i_0}{\pi a^2};\quad j_1 = \sqrt{\frac{I_1}{a^2\sigma\mu_0}};\quad B_1 = \sqrt{\frac{I_1\mu_0}{\sigma}};\quad t_1 = a^2\sigma\mu_0;$$

I_1 is the limiting value of the "current integral" [6]; σ is the conductivity of the conductor material; $\mu_0 = 4\pi\cdot10^{-7}$ G/m; r, φ is the coordinate of that point in the cylinder section at which the "current integral" reaches its limiting value; $\xi_{m,n} = a\sqrt{\gamma_{m,n}}$; $\gamma_{m,n}$ are the roots of the equations $J_1(a\sqrt{\gamma_m})=0$; $J_0(a\sqrt{\gamma_n})=0$; J_0, J_1 are the zeroth- and first-order Bessel functions; a is the conductor radius.

From the equation of motion we find for the heat-limited velocity and displacement at time τ_1

$$v^* = j_0^*\beta f_1(\tau_1,\ \tau_0,\ T_1),\quad s^* = j_0^*\beta f_2(\tau_1,\ \tau_0,\ T_1), \tag{2}$$

where

$$v^* = v/v_1;\quad s^* = s/s_1;\quad v_1 = I_1 a\mu_0/\gamma;\quad s_1 = v_1 t_1;$$

γ is the density of the conductor material;

$$f_1 = \tau_1 - \tau_0 - 2T_1\left(e^{-\frac{\tau_0}{T_1}} - e^{-\frac{\tau_1}{T_1}}\right) + T_1\left(e^{-\frac{2\tau_0}{T_1}} - e^{-\frac{2\tau_1}{T_1}}\right)\Big/2;$$

$$f_2 = (\tau_1^2-\tau_0^2)/2 + 2T_1^2\left(e^{-\frac{\tau_0}{T_1}} - e^{-\frac{\tau_1}{T_1}}\right) - T_1^2\left(e^{-\frac{2\tau_0}{T_1}} - e^{-\frac{2\tau_1}{T_1}}\right)\Big/4 - (\tau_1-\tau_0)\left(\tau_0 + 2T_1 e^{-\frac{\tau_0}{T_1}} - T_1 e^{-\frac{2\tau_0}{T_1}}\Big/2\right).$$

It follows from physical considerations that there must exist some values of accelerating magnetic field induction, field increment time constant, and current which are optimal from the viewpoint of maximum velocity. In fact, with increase in the parameter β and decrease in T_1 the currents induced in the conductor increase, which leads to intense heating and a decrease in the maximum possible velocity. At low values of β the value of the accelerating

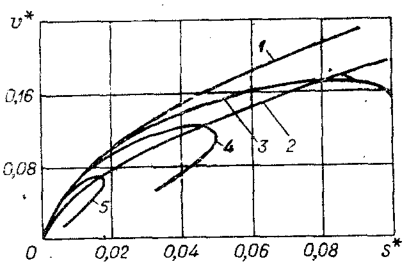

Fig. 1

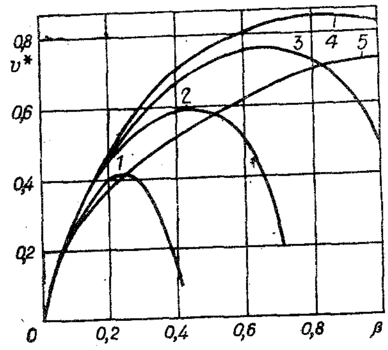

Fig. 2

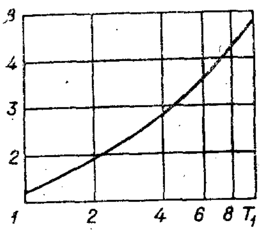

Fig. 3

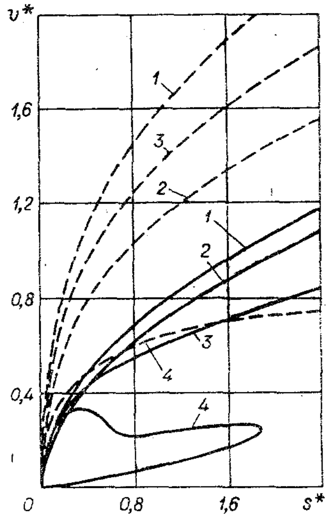

Fig. 4

force decreases, as a result of which the velocity decreases even for significant current densities. Similarly, decrease in the parameter T_1, which corresponds to high slopes in the current and field curves, intensifies the surface effect and decreases the permissible current density, and thus, the velocity. Growth in T_1 leads to expulsion of the body from the acceleration zone at the front of the accelerating force and a decrease in velocity.

Results of calculations with Eqs. (1), (2) for the point $r = a$, $\varphi = -\pi/2$, where the heating is maximum, shown in Fig. 1 ($T_1 = 5$; $\tau_0 = 0$; 1 – $\beta = 2.6$; 2 – 1; 3 – 3.8; 4 – 4.2; 5 – 5.0) and Fig. 2 ($s^* = 1.5$; $\tau_0 = 0$; 1 – $T_1 = 0$; 2 – 0.1; 3 – 0.4; 4 – 0.8; 5 – 10), confirm the mechanism of accelerating field interaction with the conductor described above. For values of the parameters β, s^*, T_1 at which the conductor heating is determined by current induced by the accelerating magnetic field, two acceleration regimes are possible (two values of the maximum heat-limited current density): a "rapid" mode characterized by higher velocity, and a "slow" mode, with longer acceleration time, and lower current density and velocity. Due to the strong effect of induced currents, the velocity does not increase monotonically as the acceleration path is increased, and there is a velocity maximum (Fig. 1).

With decrease in induced currents and increase in the parameter T_1 and decrease in β the velocity rises monotonically with increase in acceleration path length. The limiting values of T_1 and β, at which a velocity maximum is absent for the case $\tau_0 = 0$ are shown in Fig. 3.

We will consider the effect of the initial phase (τ_0 is the decay time) of the current in the conductor relative to the moment of accelerating field application. Introduction of the initial phase permits a separation in time of the processes of diffusion of the accelerating and intrinsic magnetic fields in the conductor, which leads to a decrease in heating and increase in maximum permissible axial current density. It then becomes possible to reduce the induced currents by increasing the parameter T_1, insuring acceleration near the induction amplitude by appropriate choice of τ_0. The calculation results of Fig. 4 (solid lines,

$\tau_0 = 0$; dashes, $\tau_0 = 16$; $T_1 = 5$; $1 - \beta = 1$; $2 - 2.4$; $3 - 3.0$; $4 - 3.3$) show a significant (by ∿100%) increase in velocity. Thus the optimum regime for electromagnetic accelerating devices is that in which there is initially a slow diffusion of the accelerating magnetic field, followed by a rapid increase in current in the conductor.

We will determine the value of the parameter β, at which the velocity is maximum for a given acceleration path. The solution of this problem reduces to finding a maximum in the function [7]

$$\Phi = v^*(\beta) + \lambda\left[s_1^* - s^*(\beta)\right],$$

where λ is an undetermined Lagrange factor; s^*_1 is the specified acceleration path in relative units.

Differentiating Φ with respect to β with consideration of Eq. (2) and setting the derivative equal to zero, after simple, but cumbersome calculations, we find

$$\beta_0 = \left[\frac{-\alpha_1\alpha_2 + \alpha_2\sqrt{\alpha_1\alpha_3}}{32\alpha_3\left(\alpha_2^2 - \alpha_1\alpha_3\right)}\right]^{1/2},$$

where α_1, α_2, α_3 are calculated just as in Eq. (1).

Analysis reveals that for $\beta = \beta_0$ over a wide range of values of the parameter T_1 (in our calculations $T_1 \in [5, 200]$) and at $\tau_0 \approx 3T_1$ the velocity changes no more than 20%.

This small variation in velocity with variation in the parameter T_1 is a consequence of the fact that for $\tau_0 = 3T_1$ the acceleration is practically completed before the induction and current attain their amplitude values. The action of the surface effect and induced currents lead to an increase in β_0 with increase in T_1 and corresponding reduction on β_0 with decrease in T_1. However for $T_1 \geq 5$ at the time when the axial current is applied to the conductor being accelerated the currents induced by the field have dampled to a significant degree. As a consequence, for the acceleration regimes considered the effect of induced currents is insignificant and the dependence of β_0 on T_1 is weak, which explains the results obtained for velocity. Thus, in practice, realization of acceleration regimes should include $\beta = \beta_0$, $T_1 \geq 5$, and $\tau_0 = 3T_1$.

In conclusion, the author considers it his pleasant duty to express his gratitude to V. N. Bondaletov for his valuable device.

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